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*A bilinear Bourgain lemma*

An important lemma due to Bourgain generalizes the ($L^2$) inequality for the Hardy-Littlewood maximal function to the case of arbitrary N frequencies. In particular, if $\{\lambda_k\}_{k=1,\dots,N}$ are $N$ distinct frequencies, one has

$$\| \sup_{0<\delta\ll 1} |\big(\sum_{k=1}^{N} \mathbf{1}_{[\lambda_k-\delta,\lambda_k+\delta]}\widehat{f}\,\big)^{\wedge}|\|_{L^2} \lesssim (\log N)^2\|f\|_{L^2}.$$

This was used by Lacey to study the non-trivial range of boundedness of the bilinear Hardy-Littlewood maximal function.

We consider bilinear versions of the above inequality and discuss their validity. In particular, when the multipliers $\mathbf{1}_{[\lambda_k-\delta,\lambda_k+\delta]}$ are replaced by sums of bilinear Hilbert transforms and one replaces the $\ell^1$ sum by $\ell^2$ (thus leading to a maximal square function), we show the resulting operator is $L^p \times L^q \to L^s$ bounded ($1/s = 1/p+1/q$) for $p, q > 2$ with constant at most logarithmic in $N$. This work is motivated by the desire to understand related bilinear square functions (in general, the goal is to understand frequency orthogonality in the bilinear setting).

This is joint work with Cristina Benea.