



## **Journées annuelles du GDR Analyse Fonctionnelle, Harmonique et Probabilités**

**October 8-10, 2018  
Laboratoire J.A. Dieudonné (LJAD), Nice**

### **Main speakers**

Franck Barthe (Univ. Paul Sabatier, Toulouse, France)  
Frédéric Bayart (Univ. Clermont Auvergne, France)  
Sophie Grivaux (Univ. Lille, France)  
Tatiana Smirnova-Nagnibeda (Univ. Genève, Switzerland)  
Emmanuel Russ (Univ. Grenoble Alpes, France)  
Stefan Wenger (Univ. Fribourg, Switzerland)

### **Short talks**

Arafat Abbar (Univ. Paris-Est, France)  
Irène Casseli (Aix-Marseille Univ., France)  
Benjamin Céleriès (Univ. Claude Bernard Lyon 1, France)  
Tonie Farès (Univ. d'Artois, France)  
Valentin Ferenczi (Univ. Sao Paulo, Brasil)  
Clifford Gilmore (Univ. Manchester, United Kingdom)  
Colin Petitjean (Univ. Bourgogne Franche Comté, France)  
Marco Vitturi (Univ. Nantes, France)

### **Scientific committee**

Franck Barthe, Gilles Godefroy, Sandrine Grellier, Sophie Grivaux, Christian Le Merdy, El-Maati Ouhabaz, Alain Valette

### **Organizing committee**

Erwann Aubry, Catalin Badea, Indira Chatterji, Séverine Rigot



## Schedule

| Monday, October 8         | Tuesday, October 9                   | Wednesday, October 10      |
|---------------------------|--------------------------------------|----------------------------|
|                           | 9:00-9:50<br>S. Wenger               | 9:00-9:50<br>F. Bayart     |
|                           | 10:00-10:40<br>C. Petitjean          | 10:00-10:40<br>T. Farès    |
|                           | Coffee break                         | Coffee break               |
|                           | 11:15-11:55<br>V. Ferenczi           | 11:15-11:55<br>B. Célariès |
|                           | Lunch                                | Lunch                      |
| 13:30-14:00 Welcome       |                                      |                            |
| 14:00-14:50<br>E. Russ    | 14:00-14:50<br>T. Smirnova-Nagnibeda |                            |
| 15:00-15:40<br>M. Vitturi | 15:00-15:40<br>C. Gilmore            |                            |
| Coffee break              | Coffee break                         |                            |
| 16:15-16:55<br>I. Casseli | 16:15-16:55<br>A. Abbar              |                            |
| 17:00-17:50<br>F. Barthe  | 17:00-17:50<br>S. Grivaux            |                            |

All talks take place in the conference room, ground floor of the LJAD.

Social dinner on Tuesday, October 9, at 20:30  
 Restaurant L'Escalinada, 22 rue Pairolière, Nice  
 Tramway station Cathédrale Vieille Ville

## Abstracts

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### Main talks

**Franck Barthe (Univ. Paul Sabatier, Toulouse)**

*Spectral gap, symmetries and log-concave perturbations*

The analytic properties of strictly log-concave probability measures are well understood, thanks to the works of Bakry and his collaborators. The case of log-concave measures -but no strictly so- is more delicate. It encompasses the case of uniform measures on compact convex sets. The Kannan-Lovász-Somonovits conjecture (KLS) predicts an approximate value of the spectral gap of these measures. In a joint work with Bo'az Klartag, which was initiated by a specific question on a measure related to the LASSO estimator, we study Poincaré inequalities for log concave perturbations of product measures. We use techniques that were devised in order to tackle the KLS conjecture, in particular in the presence of symmetries. Other key ingredients are gaussian mixtures and a recent extension of the Gaussian correlation inequality. As an application, we can confirm, up to logarithmic corrections, the KLS conjecture for sections of unit balls of  $\ell_p^n$  ( $p \in [1, 2]$ ) by vector subspaces of at least proportional dimension.

**Frédéric Bayart (Univ. Clermont Auvergne)**

*Multifractal analysis of divergence and convergence of wavelets series*

We study the convergence and divergence of the wavelet expansion of a function in a Sobolev or a Besov space from a multifractal point of view. In particular, we give an upper bound for the dimension of the set of points where the expansion converges (or diverges) at a given speed, and we show that, generically, these bounds are optimal.

**Sophie Grivaux (Univ. Lille)**

*Fourier coefficients of continuous measures on the Furstenberg sequence*

I will explain how to construct continuous probability measures on the unit circle which have the property that the modulus of their Fourier coefficients on the Furstenberg sequence  $\{2^k 3^l ; k, l \geq 1\}$  is bounded away from zero. This answers in the negative a Conjecture of R. Lyons, motivated by the Furstenberg Conjecture concerning  $\times 2$  and  $\times 3$  invariant probability measures on the circle. This is joint work with Catalin Badea (Lille).

**Tatiana Smirnova-Nagnibeda (Univ. Genève)**

*Laplacians on graphs associated with self-similar group actions*

We will discuss spectra of Laplacians on graphs and subshifts that arise naturally in the study of some interesting finitely generated groups, such as Grigorchuk's groups of intermediate growth and other self-similar groups.

**Emmanuel Russ (Univ. Grenoble Alpes)**

*A minimization problem for the principal eigenvalue of the Laplacian with large drift*

Let  $\Omega \subset \mathbb{R}^d$  be a  $C^2$  bounded domain and  $L = \Delta + v \cdot \nabla$  on  $\Omega$  under Dirichlet boundary condition, where  $v$  is a bounded vector field on  $\Omega$ . We consider the minimal principal eigenvalue  $\lambda_1$  and the related principal eigenfunction  $\varphi$  in the class of drifts having a given, but large, pointwise upper bound, and prove asymptotic properties of  $\lambda_1$  and  $\varphi$ . This is joint work with François Hamel and Luca Rossi.

**Stefan Wenger (Univ. Fribourg)**

*Isoperimetric characterization of non-positive curvature*

The aim of this talk is to show that a locally compact geodesic metric space has non-positive curvature in the sense of Alexandrov (i.e. is a CAT(0)-space) if and only if it admits a quadratic isoperimetric inequality for curves with sharp Euclidean constant, that is, if every closed curve of length  $l$  bounds a disc of area at most  $(4\pi)^{-1}l^2$ . The proof of this result is based on (1) a solution of the classical problem of Plateau in the general setting of proper metric spaces and (2) properties of the intrinsic structure of minimal discs in metric spaces. Based on joint work with A. Lytchak.

## Short talks

**Arafat Abbar (Univ. Paris-Est)**

*$\Gamma$ -supercyclicity for  $c_0$ -semigroups*

For  $\Gamma$  a subset of  $\mathbb{C}$ , a bounded linear operator  $T$  on a Banach space  $X$  is said to be  $\Gamma$ -supercyclic if there is a vector  $x$  in  $X$  such that  $\text{Orb}(\Gamma x, T) := \{\lambda T^n x : \lambda \in \Gamma, n \in \mathbb{N}\}$  is dense in  $X$ . S. Charpentier, R. Ernst and Q. Menet characterized the sets  $\Gamma \subset \mathbb{C}$  for which  $\Gamma$ -supercyclicity implies hypercyclicity. They also characterized the set  $\Gamma$  for which, if  $\text{Orb}(\Gamma x, T)$  is somewhere dense, then  $x$  is a hypercyclic vector for  $T$ . In this talk, we will be interested in versions of these results for  $c_0$ -semigroups.

**Irène Casseli (Aix-Marseille Univ.)**

*Sarason's problem in polyanalytic Fock spaces*

Polyentire functions generalize entire functions in that they are solutions of "Cauchy-Riemann equations of order  $n$ ", of the form  $\partial^n f / \partial \bar{z}^n = 0$ , over the whole complex plane. Polyanalytic Fock space  $F_n^2$  is, by analogy with the classical case, the closed subspace of the Hilbert space  $L^2(\mathbb{C}, d\mu)$ , where  $\mu$  is the Gaussian probability measure over  $\mathbb{C}$ , of polyentire functions of order  $n$ ; given a function  $f$ , the Toeplitz operator with symbol  $f$  is formally defined by  $T_f^n(h) = P_{F_n^2}(fh)$ , where  $P_{F_n^2}$  is the orthogonal projection from  $L^2(\mathbb{C}, d\mu)$  onto  $F_n^2$ . The so-called Sarason's problem emerged from the context of the classical Hardy and Bergman spaces of the unit disk  $\mathbb{D}$ . It consists in finding necessary and sufficient conditions on the symbols  $f$  and  $g$  for the Toeplitz product with symbols  $f$  and  $\bar{g}$  to be bounded in the Fock space. In this talk, we will first review some known results on the classical Fock space; then I will present some generalizations of these results for polyanalytic Fock spaces. This work is part of my PhD thesis.

**Benjamin Céleriès (Univ. Claude Bernard Lyon 1)**

*On the hidden point spectrum of composition operators*

Let  $Hol(D)$  be the space of holomorphic functions on the open unit disk  $D$  of the complex plane. Let  $X$  be an arbitrary Banach space which embeds continuously in  $Hol(D)$ . The aim of this talk is to describe the spectral projections on the eigenspaces of  $C_\varphi$ , where  $\varphi : D \rightarrow D$  is holomorphic,  $\varphi$  has a fixed point in  $D$ , and  $C_\varphi \in \mathcal{L}(X)$  is given by  $C_\varphi(f) = f \circ \varphi$ . The main interest of our result is that it is still valid for eigenspaces associated with non-isolated eigenspaces.

**Tonie Farès (Univ. d'Artois)**

*Absolutely summing and nuclear composition operator on the Bloch space with contact points*

We are going to give a sufficient condition for a composition operator  $C_\Phi(f) = f \circ \Phi$  to be  $p$ -summing on the Bloch space  $B$  (the space of analytic functions  $f$  on the unit disc  $D$  which satisfy  $\sup(1 - |z|^2)|f'(z)|$  is finite). We construct examples of a conformal mapping of the unit disc  $D$  into itself with contact points with the unit circle  $T$ , which induce  $p$ -summing composition operators. We also construct such an example inducing a nuclear composition operator on  $B$ . We finish by giving a characterization for a composition operator to be nuclear on the Bloch space  $B$ .

**Valentin Ferenczi (Univ. São Paulo)**

*On homogeneity and Ramsey properties of  $L_p$ -spaces*

We shall present new results on the approximate homogeneity of the classical  $L_p$ -spaces,  $1 \leq p < +\infty$ , or equivalently on the approximate transitivity of the action of the isometry groups of those spaces. We shall indicate a manner in which Fraïssé theory may be developed in this context to claim that  $L_p$  is the Fraïssé limit of (some class of) its subspaces, and shall discuss the difference between the case  $p = 4, 6, 8, \dots$  and the other case. Finally we shall explain how to recover from this the extreme amenability of the isometry group of  $L_p$  (proved by Gromov-Milman for  $p = 2$  and Giordano-Pestov for the other values). Joint work with J. Lopez-Abad, B. Mbombo, S. Todorčević.

**Clifford Gilmore (Univ. Manchester)**

*Hypercyclicity of Derivations*

We examine the hypercyclicity of generalised derivations  $S \mapsto AS - SB$ , for fixed bounded linear operators  $A, B$ , on spaces of operators. Hitherto the principal result in this setting has been the characterisation of the hypercyclicity of the left and right multiplication operators by Bonnet, Martínez-Giménez and Peris (2004). The main example I will show is the existence of non-trivial hypercyclic generalised derivations on separable ideals of operators. I will also outline joint work with Saksman and Tylli, which gives the somewhat surprising result that scalar multiples of the backward shift operator  $cB$  never induce hypercyclic commutator maps  $S \mapsto c(BS - SB)$  on separable ideals of operators on  $\ell^2$ .

**Colin Petitjean (Univ. Bourgogne Franche-Comté)**

*On Lipschitz maps which attain their norm*

Let  $X$  be a Banach space and  $M$  be a metric space equipped with a distinguished point denoted  $0$ . We consider  $Lip_0(M, X)$  the space of Lipschitz maps  $f : M \rightarrow X$  which satisfy  $f(0) = 0$ . Equipped with the norm  $\|f\|_L$ , being the best Lipschitz constant of  $f$ ,  $Lip_0(M, X)$  is a Banach space. We then say that a Lipschitz map  $f \in Lip_0(M, X)$  strongly attains its norm whenever there is  $x \neq y \in M$  such that  $\|f(x) - f(y)\|_X = \|f\|_L d(x, y)$ . Next, there is a different notion of norm attainment. It is known that there is a Banach space  $\mathcal{F}(M)$  together with an isometry  $\delta : M \rightarrow \mathcal{F}(M)$  such that every  $f \in Lip_0(M, X)$  extends uniquely to a continuous operator  $\bar{f} \in \mathcal{L}(\mathcal{F}(M), X)$  satisfying  $\|\bar{f}\| = \|f\|_L$  and  $\bar{f} \circ \delta = f$ . The Banach space  $\mathcal{F}(M)$  is the so called Lipschitz free space over  $M$ . We now say that a Lipschitz map  $f \in Lip_0(M, X)$  attains its operator norm if there exists an element  $\gamma \in \mathcal{F}(M)$  such that  $\bar{f}(\gamma) = \|f\|_L$  and  $\|\gamma\|_{\mathcal{F}(M)} = 1$ . We will analyze the relationships between the above-mentioned notions of norm attainment. This will lead us quite naturally to the study of the extremal structure of Lipschitz free spaces. In light of the celebrated Bishop-Phelps theorem, we will also analyze when the class of Lipschitz functions which strongly attain their norm is dense in  $Lip_0(M, X)$ .

**Marco Vitturi (Univ. Nantes)**

*A bilinear Bourgain lemma*

An important lemma due to Bourgain generalizes the  $(L^2)$  inequality for the Hardy-Littlewood maximal function to the case of arbitrary  $N$  frequencies. In particular, if  $\{\lambda_k\}_{k=1,\dots,N}$  are  $N$  distinct frequencies, one has

$$\| \sup_{0 < \delta \ll 1} | \left( \sum_{k=1}^N \mathbf{1}_{[\lambda_k - \delta, \lambda_k + \delta]} \widehat{f} \right)^\wedge | \|_{L^2} \lesssim (\log N)^2 \|f\|_{L^2}.$$

This was used by Lacey to study the non-trivial range of boundedness of the bilinear Hardy-Littlewood maximal function. We consider bilinear versions of the above inequality and discuss their validity. In particular, when the multipliers  $\mathbf{1}_{[\lambda_k - \delta, \lambda_k + \delta]}$  are replaced by sums of bilinear Hilbert transforms and one replaces the  $\ell^1$  sum by  $\ell^2$  (thus leading to a maximal square function), we show the resulting operator is  $L^p \times L^q \rightarrow L^s$  bounded ( $1/s = 1/p + 1/q$ ) for  $p, q > 2$  with constant at most logarithmic in  $N$ . This work is motivated by the desire to understand related bilinear square functions (in general, the goal is to understand frequency orthogonality in the bilinear setting). This is joint work with Cristina Benea.